

Matroidal Degree-Bounded Minimum Spanning Trees

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MIT

Degree-constrained MST problems

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Types of degree constraints

- Degree-bounded MST (the classic):

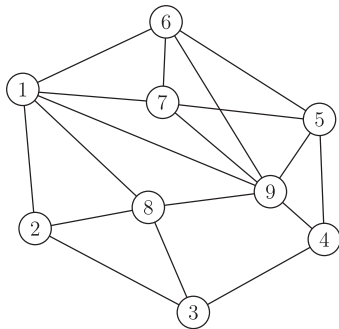
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- Cut-constraints:

$$|T \cap \delta(S)| \leq B_S \text{ for } S \in \mathcal{S} \subseteq 2^V.$$

- 0/1-packing constraints:

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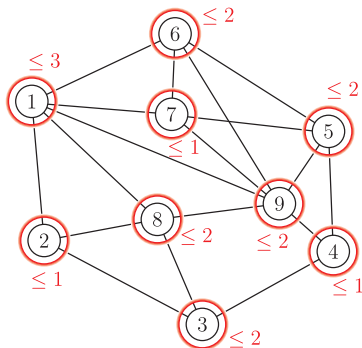
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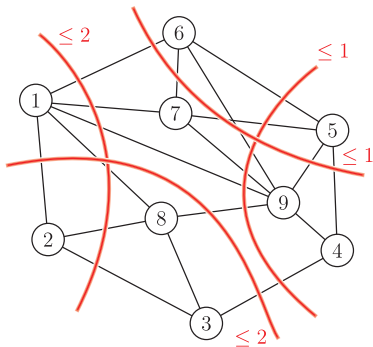
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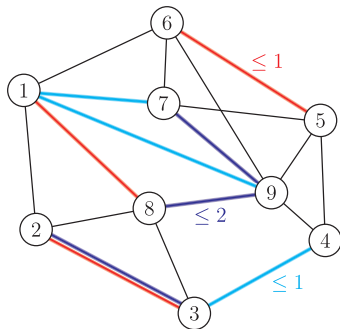
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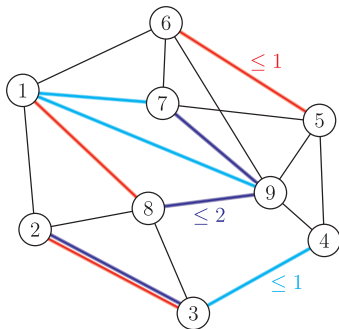
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Typical motivations for degree constraints

- Technical restrictions (VLSI design, telecommunication networks).
- Increase reliability by avoiding overloaded vertices.
- Reduce vulnerability against malicious attacks.

Trading slight infeasibility for “optimal” cost

At the example of degree-bounded MST

$$OPT = \min\{c(T) \mid T \in \underbrace{\mathcal{T}}, |T \cap \delta(v)| \leq B_v \forall v \in V\}$$

all spanning trees ($\subseteq 2^E$)

- Even checking feasibility is NP hard
(e.g. if $B_v = 2 \forall v \in V \rightarrow$ Hamiltonian path problem.)

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- $OPT_f = \min c^T x$
 $x \in P_{ST} = \text{conv}(\{\mathbf{1}_T \mid T \in \mathcal{T}\})$
 $x(\delta(v)) \leq B_v \quad \forall v \in V$
 ~~$x \in \{0,1\}^E$~~
- $\min\{c(T) \mid T \in \mathcal{T}, |T \cap \delta(v)| \leq B_v + k \forall v \in V\}$

Previous results

Degree-bounded MST

- **+1** violation (**no costs**) Furer and Raghavachari (1994)
- various **super-constant** violations with **cost** $\leq OPT_f$ Könemann and Ravi (2002, 2003), Chaudhuri et al. (2005), ...

- **+2** Goemans (2006)
- **+1** Singh and Lau (2007)

Generalized bounds

Laminar cut bounds:

- **$+O(\log |V|)$** Bansal et al. (2010)

Bounds on arbitrary edge sets $x(U) \leq B_U \forall U \in \mathcal{U} \subseteq 2^E$:

- **$+\max_{e \in E} |\{U \in \mathcal{U} \mid e \in U\}|$** Bansal et al. (2009)

Thinness bounds wrt $y \in P_{ST}$: $x(\delta(S)) \leq y(\delta(S)) \forall \emptyset \neq S \subsetneq V$ (*thin trees*):

- **$\times O\left(\frac{\log |V|}{\log \log |V|}\right)$** Asadpour et al. (2010)

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Obtain constant violation for constraints beyond degree-bounded MST?

Our contributions

Main results

- Efficient algorithm with $+8$ guarantee for **matroidal degree constraints**.
- This is based on **extensions to the iterative relaxation framework**.

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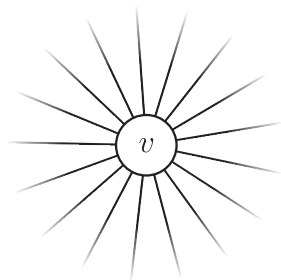
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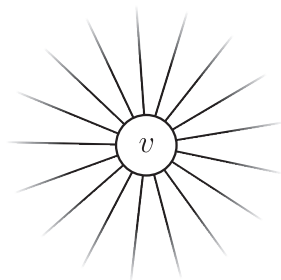
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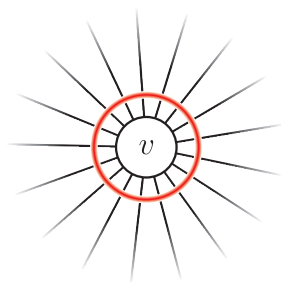
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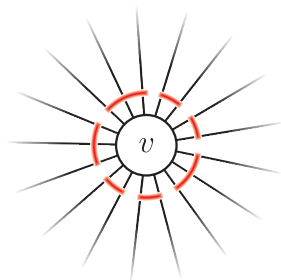
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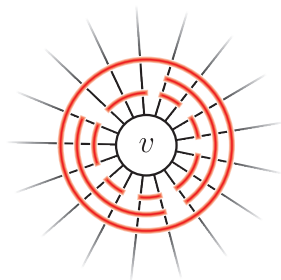
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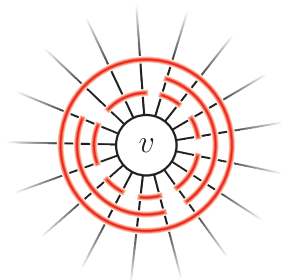
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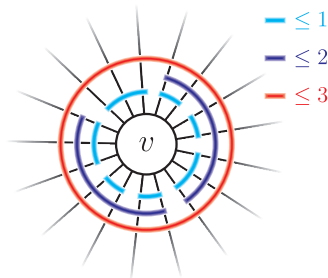
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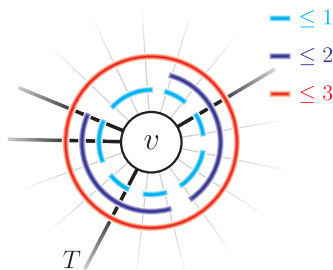
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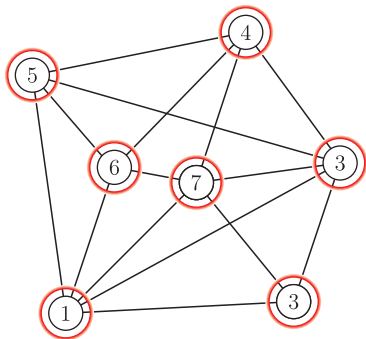
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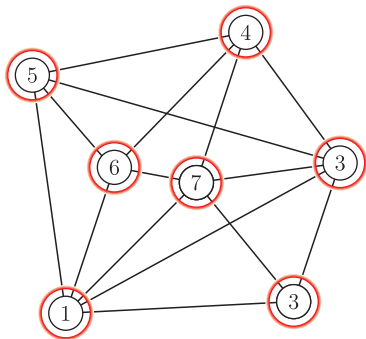
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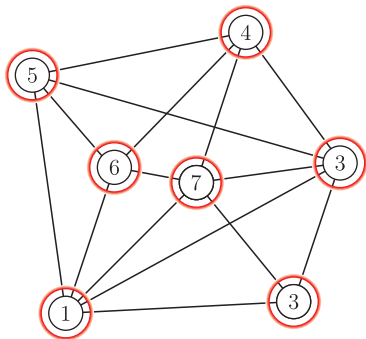
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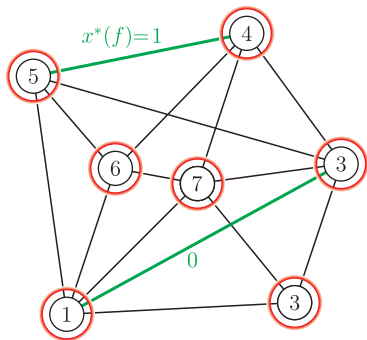
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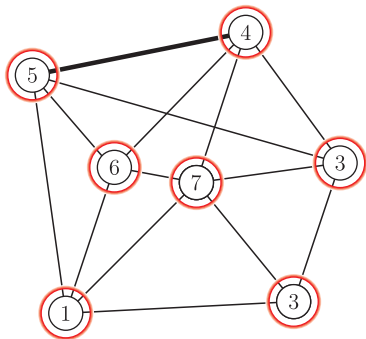
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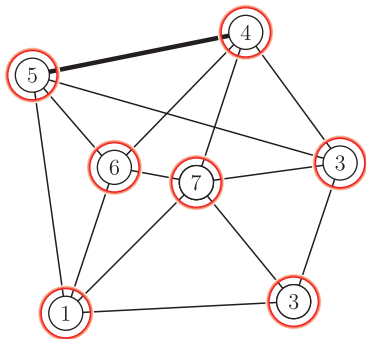
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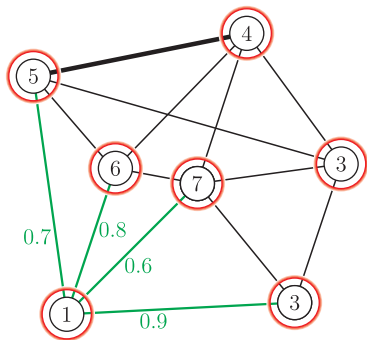
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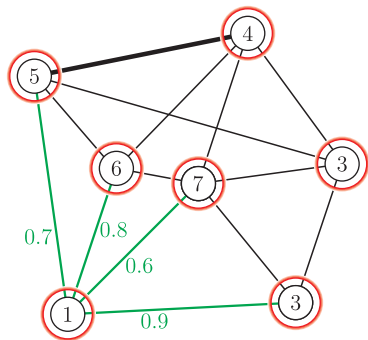
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$$z(\delta(1)) = 0.3 + 0.2 + 0.4 + 0.1 = 1 < 2$$

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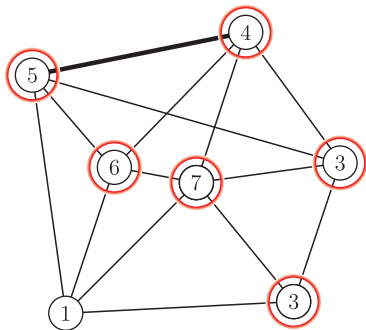
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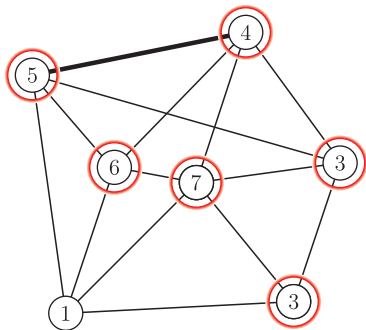
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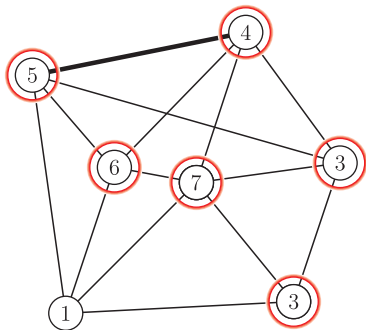
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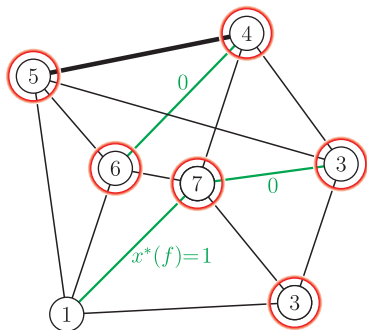
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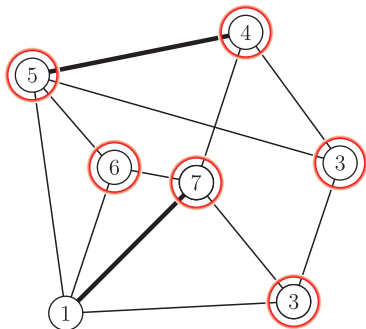
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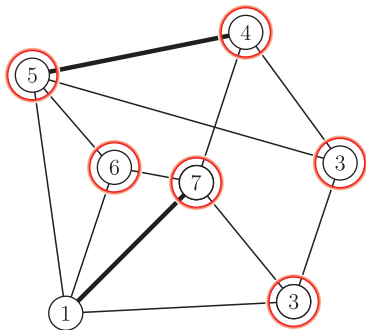
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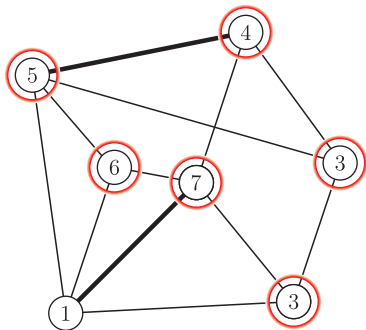
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Key property: x^* is sparse (in particular $|\text{supp}(x^*)| \leq 2|V| - 1$).

$$\begin{pmatrix} A_{ST} \\ \hline A_{deg} \end{pmatrix} x^* \leq \begin{pmatrix} b_{ST} \\ \hline b_{deg} \end{pmatrix}$$
$$x^* \geq 0$$

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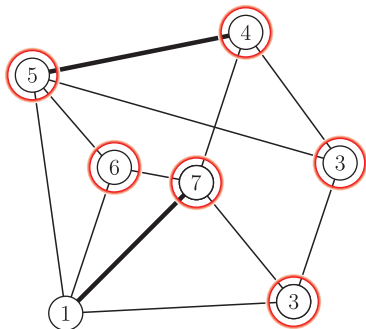
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$$\begin{pmatrix} A_{ST} \\ \text{-----} \\ A_{deg} \\ \text{-----} \\ -I \end{pmatrix} x^* \leq \begin{pmatrix} b_{ST} \\ \text{-----} \\ b_{deg} \\ \text{-----} \\ 0 \end{pmatrix}$$

Iterative relaxation by Singh and Lau (+1 guarantee)

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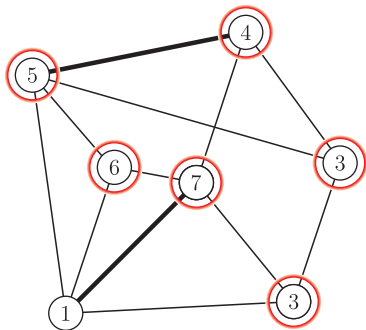
- Get basic LP solution x^* .
- If sol is integral: stop.

Step 2

- Delete 0-edges.
- Fix 1-edges.

Step 3 (let *spare*: $z = 1 - x^*$)

- Find $v \in V$ s.t. $z(\delta(v)) < 2$.
- Delete deg constraint at v .
- Back to Step 1.



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Challenges with more general/matroidal constraints

Previous iterative relaxation/rounding approaches are hard to generalize to matroidal deg constraints (or other generalized constraint).

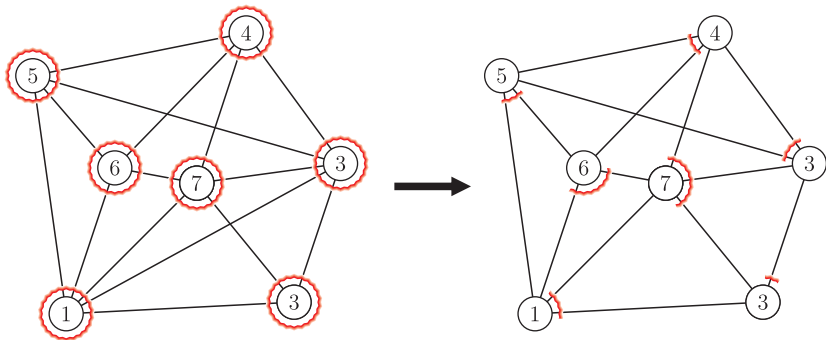
Some issues with previous iterative relaxation approaches

- Not sufficient sparsity to drop full degree constraints at some vertex.
- Previous approaches relied on the fact that each edge is only in a constant number of linear constraints (belonging to degree constraints).

High-level goal of our algorithm

Iteratively change constraints to **approach matroid intersection** problem instead of targeting ST polytope (which is a matroid base polytope).

→ Iteratively “remove” each edge $\{u, v\}$ either from deg constraint at u or v .
(this is similar in spirit to Goemans’ algorithm, but works iteratively.)



- If each edge belong to at most one degree constraints, all matroidal degree constraints together form one single matroid.
- Resulting optimization problem is matroid intersection and thus integral.

Summary of further technical contributions

Further contributions on algorithm design level

- When removing edges from constraints: old constraint gets **replaced** by a possibly more complicated matroidal constraint (s.t. violation is bounded by slack).
- We **fix tight ST constraints** → they help respect degree constraints.

New ideas for the analysis

- **New argument to prove sparsity** that exploits interplay of constraints.
- **Exploit properties of low-dimensional faces of ST polytope** (to deal with cases where many lin indep ST constraints are tight).

An oversimplified sketch of the algorithm

- $H = (W, F)$: current graph, $F_2 \subseteq F$: edges currently in both constraints.
- N_w : current matroidal deg constraints with corresp. matroid polytope P_{N_w} .

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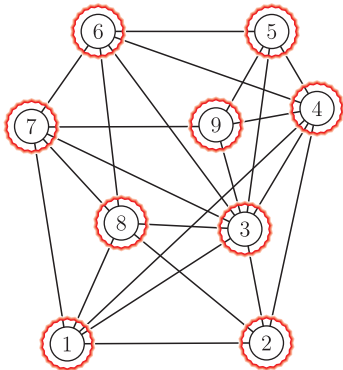
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Step 3: degree constraint adaptation

- Find $w \in W$ s.t. $z(\delta(w) \cap F_2) \leq 4$.
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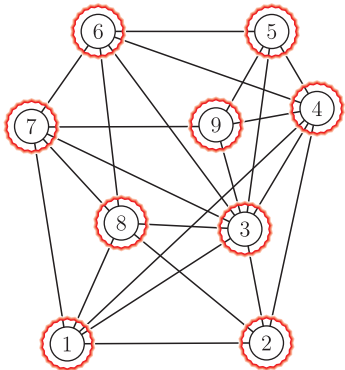
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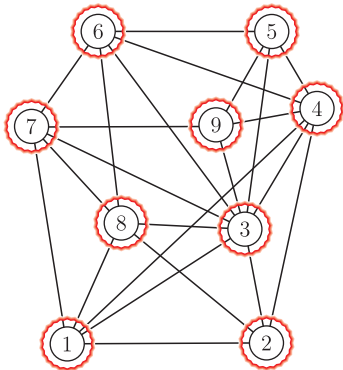
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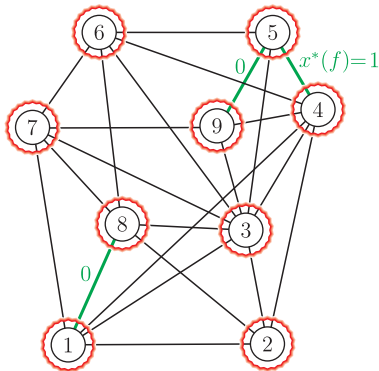
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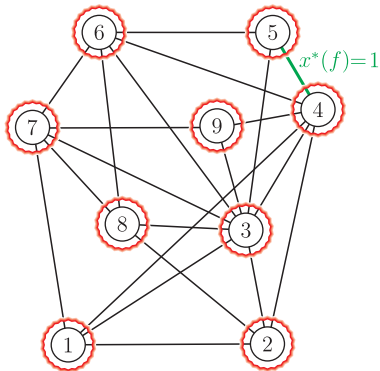
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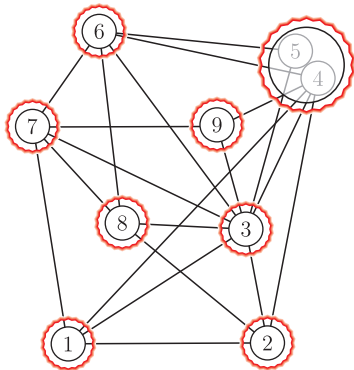
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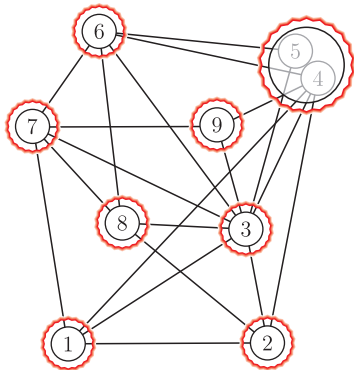
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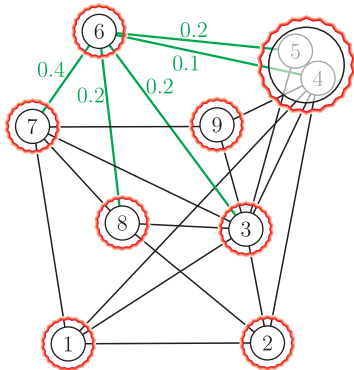
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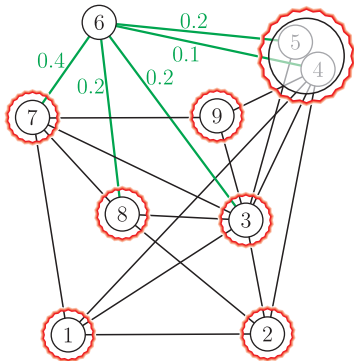
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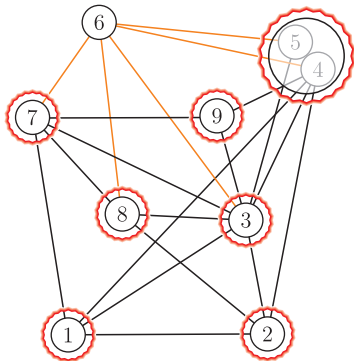
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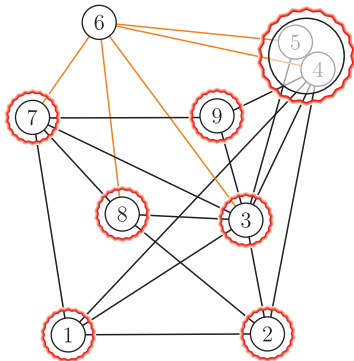
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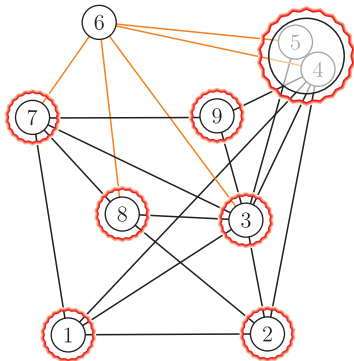
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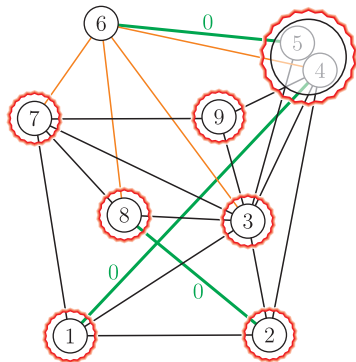
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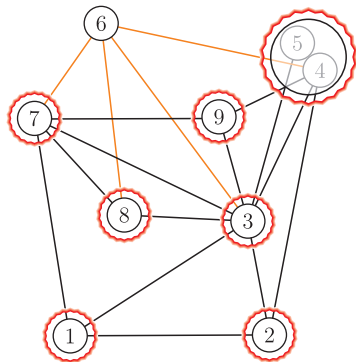
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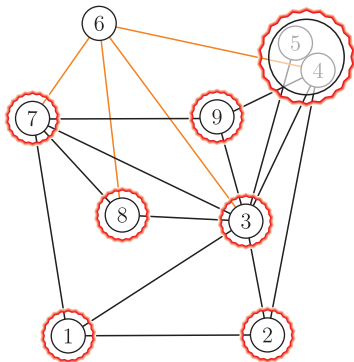
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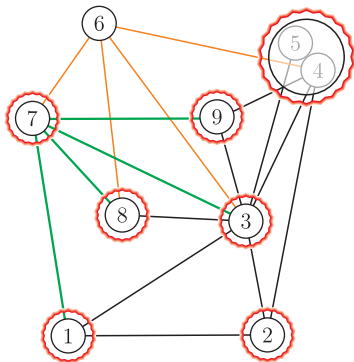
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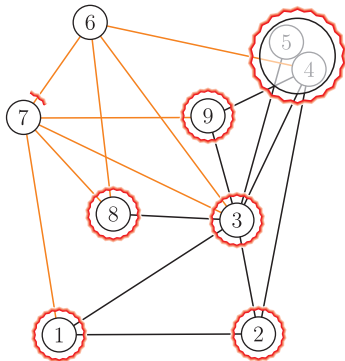
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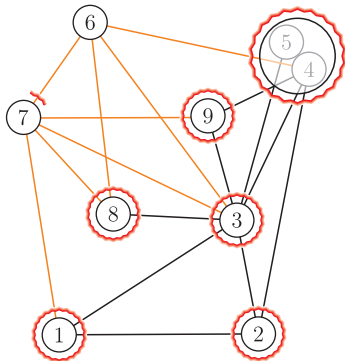
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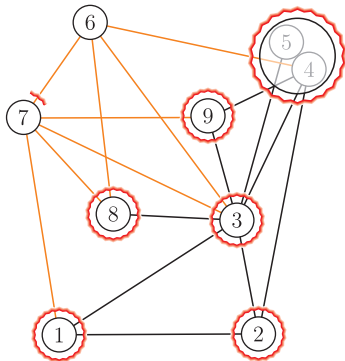
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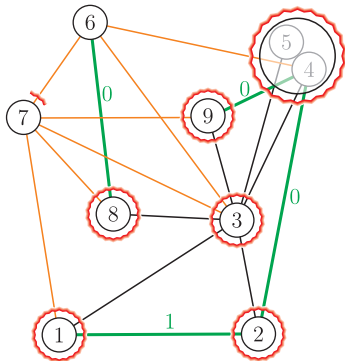
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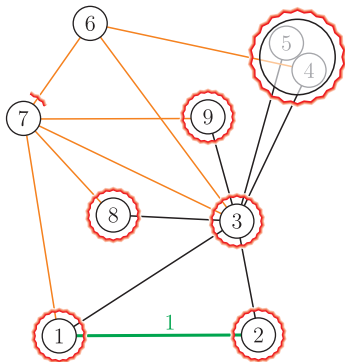
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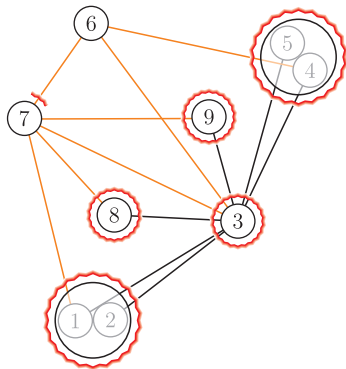
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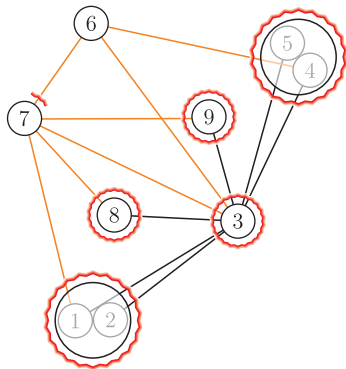
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- N_w : current matroidal deg constraints with corresp. matroid polytope P_{N_w} .

Step 1

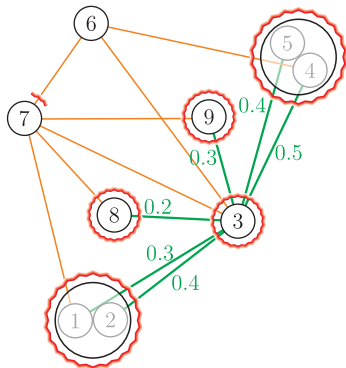
- Get basic solution x^* to LP:
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- Delete 0-edges.
- Contract 1-edges.
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Step 3: degree constraint adaptation

- Find $w \in W$ s.t. $z(\delta(w) \cap F_2) \leq 4$.
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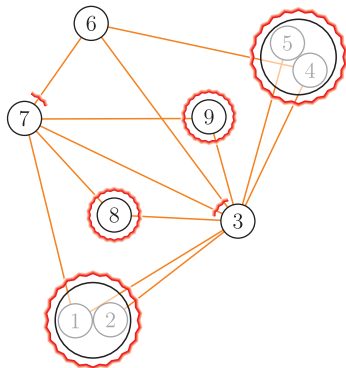
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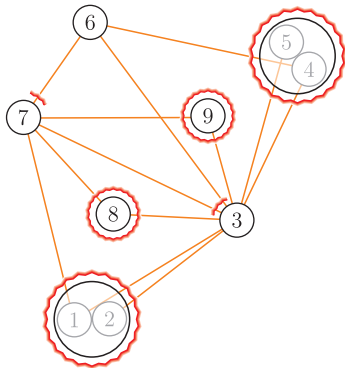
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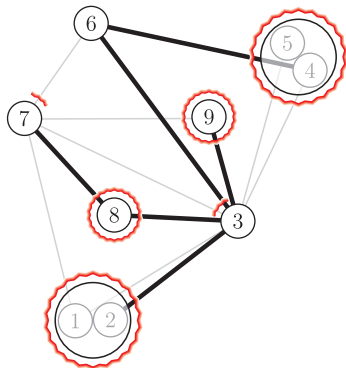
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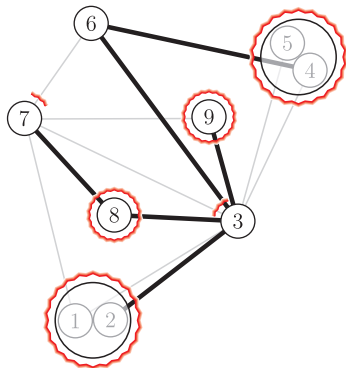
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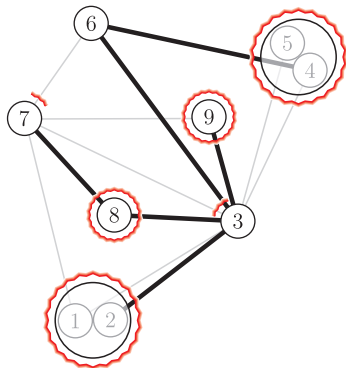
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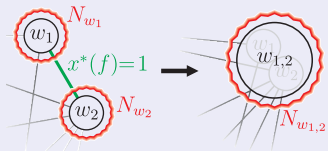
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For $v \in V$, only ≤ 1 deg adaptations impacts $\delta(v) \Rightarrow$ violation ≤ 4 .

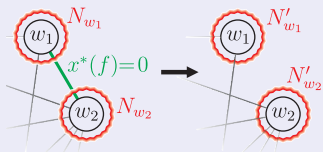
Updating degree constraints

Contraction of 1-edge



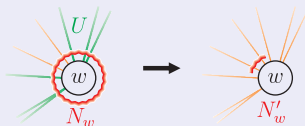
- Contract f in N_{w_1} and N_{w_2} .
- $N_{w_{1,2}}$ is disjoint union of N_{w_1} and N_{w_2} .

Deletion of 0-edge



- Delete f from N_{w_1} and N_{w_2} .

Removing edges from deg constr. (a bit more involved)



Update is done such that:

- $T \in \mathcal{T}$ satisfies $N'_w \Rightarrow T$ violates N_w by $\leq \lceil z(U) \rceil$.
- Current LP sol remains feasible.

Proving sparsity to show \exists adaptation step

- $\delta^D(w) \subseteq \delta(w)$: edges not yet removed from N_w .

Lemma

If k linearly indep degree constraints of P_{N_w} are tight wrt $x^ \Rightarrow x^*(\delta^D(w)) \geq k$.*

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- Total # of linearly indep & tight ST constraints: $|\mathcal{L}| \stackrel{\text{uncrossing}}{\leq} |W| - 1$.
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In later iterations, this averaging argument does not work anymore

- For some nodes $w \in W$ we will have $\delta^D(w) = \emptyset$.
- We improve sparsity with 2nd type of deg adaptation (\rightarrow another +4 in violation).

Conclusions

- Even for very general degree constraints (**matroidal degree constraints**), a tree of cost $\leq OPT$ can be obtained with a **constant additive degree violation**.
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Thank you!

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