

# Complex Networks – an Overview

Based on the article “The structure and function of complex networks” by M.E.J.Newman

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5.1.2006



# Overview of the presentation

## 1 Introduction

- What is a complex network?
- Complex networks in the real world
- Important properties of complex networks

## 2 Network models

- Introduction
- Models of networks with fixed size
- Models of network growth

## 3 Processes on networks

- Introduction
- Percolation theory and network resilience

# Outline

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# Typical characteristics of a complex network

There is no formal definition of complex networks. Typical characteristics of complex networks are:

- Between order and perfect randomness.
- Big.
- Dynamically changing.
- Open (difficult to find a boundary of the network).
- They are the backbone of complex systems.
- They have a long history.

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# Real-world networks as source of inspiration

The development in the domain of complex networks was heavily inspired by real-world networks:

- 1 Social networks
- 2 Information networks
- 3 Technological networks
- 4 Biological networks

# Real-world networks as source of inspiration

The development in the domain of complex networks was heavily inspired by real-world networks:

## 1 Social networks

- Collaboration networks of researchers
- Phone call networks
- Email networks

## 2 Information networks

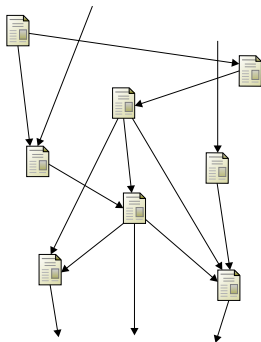
## 3 Technological networks

## 4 Biological networks

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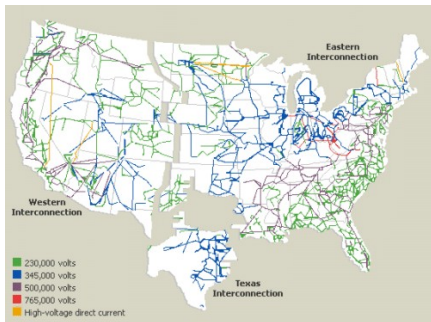
- 1 Social networks
- 2 Information networks
  - Citation networks between academic papers
  - World Wide Web
- 3 Technological networks
- 4 Biological networks



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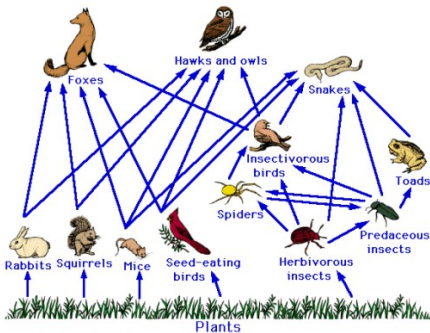
- 1 Social networks
- 2 Information networks
- 3 **Technological networks**
  - Electric power grid
  - Railways
  - Road networks
  - Internet
- 4 Biological networks



# Real-world networks as source of inspiration

The development in the domain of complex networks was heavily inspired by real-world networks:

- 1 Social networks
- 2 Information networks
- 3 Technological networks
- 4 **Biological networks**
  - Metabolic pathways
  - Food web
  - Neural networks
  - Vascular networks in plants



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# Important properties seen in complex networks

- Small-world effect
- Transitivity
- Degree distribution
- Network resilience
- Degree correlations
- Community structure and mixing patterns
- Network navigation

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How near are two arbitrarily chosen vertices in a network?

## Small-world experiment

- Letters were given to “arbitrary” persons and have to reach a certain target person.
- Every person has to pass the letter to an acquaintance.

Letters arriving to the designated target needed only about **six intermediate steps**.

# Important properties seen in complex networks

- Small-world effect
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Let  $l$  be the mean shortest distance between vertex pairs in a network.

$$l = \frac{1}{\frac{1}{2}n(n+1)} \sum_{i \geq j} d_{ij}$$

## Definition (Small-world networks)

A class of networks is said to show the small-world effect if

$$l = \mathcal{O}(\log(n))$$

# Important properties seen in complex networks

- Small-world effect
- **Transitivity**
- Degree distribution
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The friend of your friend is likely to be your friend.

## Definition (Transitivity)

Transitivity is the presence of a heightened number of triangles.

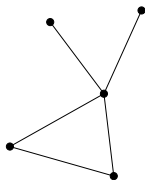
A measure of transitivity

$$C = \frac{6 \times \text{\#triangles in the network}}{\text{\#paths of length two}}$$

# Important properties seen in complex networks

- Small-world effect
- **Transitivity**
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## Example



- 16 pathes of length 2
- 1 triangle

$$C = \frac{6 \cdot 1}{16} = \frac{3}{8}$$

# Important properties seen in complex networks

- Small-world effect
- Transitivity
- Degree distribution
- Network resilience
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How many neighbors has a vertex?

Some typical degree distribution:

- Power-law (World Wide Web)
- Exponential (Power grids)

# Important properties seen in complex networks

- Small-world effect
- Transitivity
- Degree distribution
- **Network resilience**
- Degree correlations
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Is a network robust against removal of vertices?

Many real world networks have a giant component, i.e. a connected component of size  $\mathcal{O}(n)$ .

## Measuring resilience

What percentage of vertices has to be removed randomly to destroy the giant component?

# Important properties seen in complex networks

- Small-world effect
- Transitivity
- Degree distribution
- Network resilience
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Do the high-degree vertices in a network associate preferentially with other high-degree vertices?

## Measuring degree correlations

- Pearson correlation coefficient of degrees at either ends of an edge.
- Look at the mean degree of the neighbors of a vertex as a function of the degree of that vertex.

# Important properties seen in complex networks

- Small-world effect
- Transitivity
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Do the vertices belong to communities with high connectivity inside a community?

# Important properties seen in complex networks

- Small-world effect
- Transitivity
- Degree distribution
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Beginning at a vertex, how to find a specific other vertex?

The small-world experiment showed:

- There are short paths in a social network.

but also that

- People are good at finding them.

How to design networks having this property?

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# Motivation for modeling complex networks

- Why do certain networks show the small-world effect?
- Where comes the power-law degree distribution from?

The primary motivation of finding appropriate models for complex networks is to **get explanations for observed network properties**.

Two types of models:

- 1 Models where the network size is fixed.
- 2 Models adding successively vertices and edges such that the network growth.

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# Random graphs

## Poisson random graphs

Between every pair of vertices we have an edge with probability  $p$ .

- The degrees are Poisson distributed.
- This model shows the small world effect.

## A generalization: The configuration model

Random graph model allowing arbitrary degree distributions.

### Main problem

Complex networks are not perfectly random as they have

- Transitivity
- Community structure
- Degree correlations , . . .

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# Small-world model of Watts and Strogatz

How to construct a model showing the small-world effect and transitivity?

## Idea of Watts and Strogatz

Geographically near vertices are more likely to be connected.

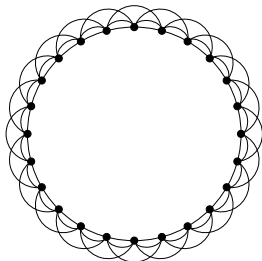
→ Transitivity

Some remote vertex pairs have to be connected.

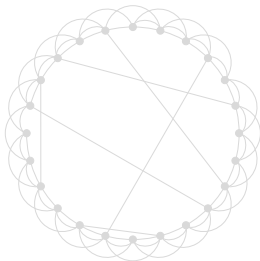
→ Small-world effect

# Small-world model of Watts and Strogatz

## Construction



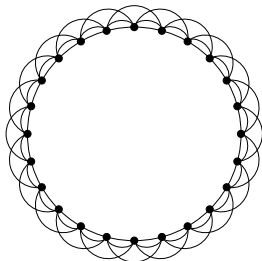
**1.Step:** Regular ring where every vertex is connected to its  $k$  nearest neighbors on both sides.



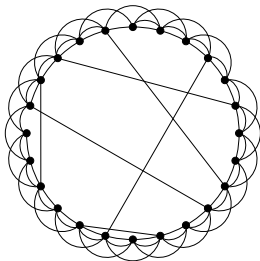
**2.Step:** A little fraction  $p$  of edges is randomly added and rewired.

# Small-world model of Watts and Strogatz

## Construction



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# Model of Barabási and Albert

## Idea of preferential attachment

New vertices prefer to attach to vertices with high degree.

## Construction

We begin with some basic network and grow the network by repeating:

- A new isolated vertex  $v$  is added to the network.
- $m$  new edges are added to the network in the following way:
  - One end is  $v$ .
  - The other end is chosen randomly from the remaining vertices such that a vertex with degree  $k$  has a probability proportional to  $k$  to get connected to the new vertex.

# Model of Barabási and Albert

## Properties

- The model shows the small-world effect.
- We have **Power-law degree distribution**.

Thanks to analysis on these kind of models, the principle of preferential attachment is believed to be the main reason for the various power-law degree distributions observed in real-world networks.

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# Motivation

The ultimate goal in the study of structures of complex networks is to understand and explain processes.

- How does the topology of the World Wide Web affect surfing?
- Which power-grid topologies keep power-failures small?
- How does the structure of a food web affect population dynamics?

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# Percolation processes

## Definition (Percolation process)

Vertices and/or edges are randomly designated either “occupied” or “unoccupied”. The focus of analysis lies in the structure of the resulting pattern of vertices.

## Examples

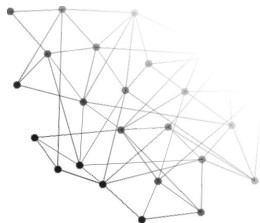
- Resistance of power-grids against cascading failures.
- Vaccination of a population against a disease.
- Robustness of the World Wide Web against failures of servers and connections.

## Example: Network resilience on networks with power-law degree distribution

- Networks with power-law degree distribution are highly robust against random removal of vertices.
- But only a little fraction (about 1-3 percent) of vertices with high degrees has to be removed to destroy the giant component.

## Summary

- Many real world networks have a complexity and uncertainty that makes it difficult to attack problems on these networks with classical techniques.
- The focus in analyzing complex networks is set on global structures (and not in local properties).
- By exploiting network structures, interesting results can be obtained concerning processes.



Thank you for your attention.

End of presentation